In: Proceedings of Institute of Electrical and Electronic Engineers, NECEC '90, Newfoundland Electrical and Computer Engineering Conference, "An Electrical Engineering Day". St. John's, NF, May 4, 1990, Session 2A.
C-CORE Publication No. 90-7.

Extraction of Ocean Wave Parameters from Wide Beam HF Radar Backscatter – A Simulation

E. W. Gill, MUN Faculty of Engineering and Applied Science/C-CORE

J. Walsh, MUN Faculty of Engineering and Applied Science

Abstract

Over the last decade, high-frequency ground wave Doppler radar has emerged as a powerful tool in the remote sensing of ocean surface phenomena, particularly in coastal regions. In this work, we examine how a four-element square array, now used by the remote sensing group at Memorial's Centre for Cold Ocean Resources Engineering for the mapping of surface currents, may be employed in the future to also extract ocean wave parameters.

I. Introduction

A Doppler radar spectrum of HF radiation backscattered from the ocean surface is generally characterized by two strong peaks appearing above and below the carrier frequency (see Fig. 1).

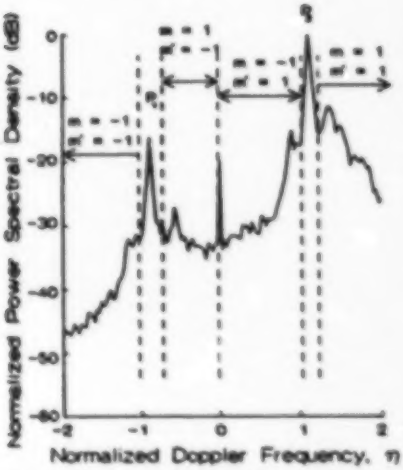

Fig. 1 A 40-sample averaged 4-element square array power spectrum normalized to its peak. Frequency is normalized to the Bragg. The m and m' are as in the text.

Crombie [1] experimentally deduced that the physical mechanism producing this phenomenon is Bragg scatter.These 'Bragg' peaks represent, for grazing incidence, radiation being reflected by ocean waves travelling towards or away from the radar and having wavelength one half that of the transmitted signal. The 'first-order' peaks are surrounded by a higher-order continuum that contains the bulk of the ocean spectral information. The second-order arises from wave-wave interactions as well as from the transmitted signal being backscattered from pairs of waves travelling perpendicularly to each other.

From the general approach of a Fourier basis expansion of the broad beam radar cross sections advanced in [2], we show how a four-element square array can be used to obtain ocean wave parameters.

The ocean is taken to be of constant, but arbitrary, depth and homogenous in its wave spectral characteristics.No attempt is made to elaborate on the effects of noise, ocean currents, or third-order wave-wave interactions on the radar spectrum.

II. Formulation of the Fourier Coefficients of the Four-element Square Array Cross Sections of the Ocean Surface

Given in [2], the wide beam radar cross section, $\sigma_j(\omega, \psi)$, at scan angle ψ may be considered as the integration, over narrow beam look direction, ϕ, of the product of the narrow beam cross section ($\sigma_j(\omega, \phi, d)$) and the power pattern, $G(\psi, \phi)$, of the wide beam array. To simplify subsequent calculations, ψ and ϕ are measured with respect to the 2–4 axis of the array. Referring to Figure 2, we write for water of fixed depth d,

$$\sigma_j(\omega, \psi) = \frac{1}{2\gamma} \int_{-\gamma}^{\gamma} G(\psi, \phi)\, \sigma_j(\omega, \phi, d)\, d\phi \,. \quad (1)$$

Here, ω is the Doppler shift in the radar spectrum and d is the water depth. The quantity 2γ is the total angle of sea surface subtended by the coastline for a given radar range cell. When $j = 1$, the first-order narrow beam cross section of

(1), which has practically no depth dependence (see [3]), may be written, in the absence of ocean currents, as

$$\sigma_1(\omega,\phi) = 2^6\pi k_0^4 \sum_{m'=\pm1} S(-2m'\vec{k_0})\delta(\omega - m'\omega_B). \tag{2}$$

Here m' signifies the sign of the Doppler shift, ω, in the radar spectrum; $\vec{k_0}$ is the radar wave vector pointing in the direction ϕ; $S(-2m'\vec{k_0})$ is the directional waveheight spectrum of the Bragg wave vectors $\pm2\vec{k_0}$. The delta-function constraint, $\delta(\cdot)$, representing the Bragg scatter condition, shows that, ideally, the first-order spectral peaks are impulse functions occurring at the Bragg frequencies $\pm\omega_B$. The broadening of these spectral peaks, as in Fig. 1, by such factors as currents and system effects is addressed by Gill [4].

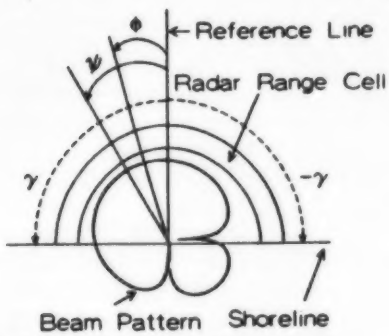

Fig. 2 Operation of a 4-element square array from a straight shore showing a typical range cell. Angles are as discussed in the text.

In equation (1), it may be deduced from [3] that for $j = 2$

$$\sigma_2(\omega,\phi,d) = 2^7\pi k_0^4 \textstyle\sum_{m'=\pm1} \int_0^\infty \int_{-\theta_0}^{\theta_0} |\Gamma|^2 \cdot S(m\vec{k})S(m'\vec{k'})\delta(\omega - m\omega_W - m'\omega'_W)\, k\, dk\, d\theta\ . \tag{3}$$

In order for second-order backscatter to occur, the wave vectors, $\vec{k}$ and $\vec{k'}$, responsible for the scattering obey the constraint as in [3]

$$\vec{k} + \vec{k'} = -2\vec{k_0}\ , \tag{4}$$

while the delta-function imposes the necessary conditions on the frequencies. Here $\vec{k'} \geq \vec{k}$ and the S's are directional waveheight spectra. The coupling coefficient, Γ, represents the second-order interactions of the scattering waves with each other as well as with the incident radiation – see [3]. The values of m and m' (i. e. ±1) – see Figure 1 – incorporate the four possible directional combinations of the scattering waves, while θ gives the angle, relative to $\vec{k_0}$, of the scatterer whose wavenumber is k. The integration limits, $\pm\theta_0$, for the ocean wavenumbers of interest can be shown to be $\pm\pi$. The quantities ω_W and ω'_W are angular frequencies corresponding to the wave vectors $\vec{k}$ and $\vec{k'}$ respectively.

The basic task confronted here is that of *inverting* (1) to give the directional ocean wave spectrum, $S(\vec{k})$, of the $\sigma_2(\omega,\phi,d)$ integral. To facilitate this, Gill [4] writes Fourier series expansions for the beam pattern, $G(\psi,\phi)$, and the directional spectra $S(k)$ and $S(k')$ appearing in (1). This can be shown to yield Fourier coefficients, $b_t(\omega)$, of the four-element radar cross sections as

$$b_t(\omega) = \frac{\pi}{\gamma}\int_{-\gamma}^{\gamma}\sum_{p=0}^{4}[(g_{tpc}\cos(p\phi) + g_{tps}\sin(p\phi))]\cdot\sigma_j(\omega,\phi,d)\,d\phi \tag{5}$$

or

$$b_t(\omega) = \frac{2}{\epsilon_t}\int_{-\pi}^{\pi}\sigma_j(\omega,\psi)tf_t(\psi)\,d\psi\ . \tag{6}$$

The range of t is from -3 to 4. The g's are Fourier coefficients of the beam pattern (see [4]) with $g_{tpc} = 0$ for $t < 0$ and $g_{tps} = 0$ for $t \geq 0$. Also, $tf_t(\psi) = \cos(t\psi)$ or $\sin(t\psi)$ for $t \geq 0$ or $t < 0$ respectively.

It is shown in [4] how $\sigma_j(\omega,\psi)$ may be created from the radar data. Once this wide beam cross section is developed, the radar spectral coefficients, $b_t(\omega)$, can be obtained numerically from (6). Then (5) is *inverted*, to yield the Fourier coefficients of the ocean wave spectrum.

III. The Integral Inversion

The integrand of (5) is nonlinear in the unknowns – i. e. in the Fourier coefficients of the ocean spectrum. Gill [4] shows how the first-order return is used to linearize the integral. Using the idea of wavenumber 'bands' over which the values of the ocean spectral coefficients are approximately constant, equation (5) is reduced to a matrix system of the form

$$\underline{A}\cdot\underline{X} = \underline{B}\ . \tag{7}$$

Here, $\underline{B}$ is a column vector of radar spectral coefficients, $\underline{A}$ is a matrix whose elements come from the summations and integrations in equation (5), while $\underline{X}$ is the column vector of ocean spectral coefficients whose values are desired.

Matrix $\underline{A}$ is, in general, singular (see [4]) and we solve the system of equation (7) using a singular value decomposition, [5].

IV. Simulation Results

To test the algorithm, simulated radar spectral coefficients were developed from (5) assuming a non-directional Pierson-Moskowitz spectrum multiplied by a cardiod directional factor as an ocean model (see [4]).

Figure 3 shows examples of ocean spectra obtained from simulated radar data. The normalized spectra shown are obtained from $S(\cdot)$ by multiplying the latter by $(2k_0)^4$.

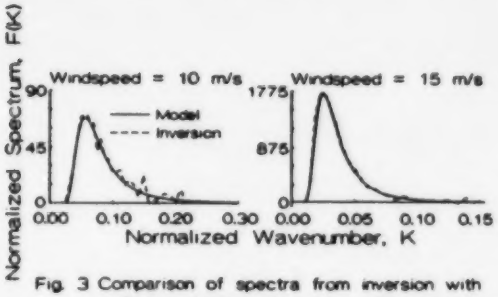

Fig. 3 Comparison of spectra from inversion with a Pierson-Moskowitz model. The normalization is done by dividing K by $2k_0$ and multiplying S(k) by $(2k_0)^4$.

By defining normalized *rms* waveheight as $H = 2k_0h$ where h is the *rms* waveheight, one may write from [4] that

$$H^2 = (2k_0)^2 \int_0^\infty k a_0(k)\, dk \qquad (8)$$

where $a_0(k)$ is the zero-order Fourier ocean spectral coefficient. Table 1 gives values of H calculated directly from a Pierson-Moskowitz ocean wave model with those extracted from simulated radar spectral coefficients through the inversion process.

Table 1 Normalized RMS Waveheight

Windspeed (m/s)	Wave Direction (deg)	Height, H from model	Height, H from inversion	% Difference in H
10	0	0.612	0.626	2.29
10	30	0.612	0.614	0.33
10	45	0.612	0.628	2.61
10	90	0.612	0.628	2.61
15	0	1.422	1.408	0.99
15	30	1.422	1.407	1.06
15	45	1.422	1.408	0.99
15	90	1.422	1.409	0.91

Figure 4 shows typical outcomes for dominant direction of the wavefield obtained from simulated data. The results are without significant error except in the tails of the spectra where oscillatory features produce some spurious angles.

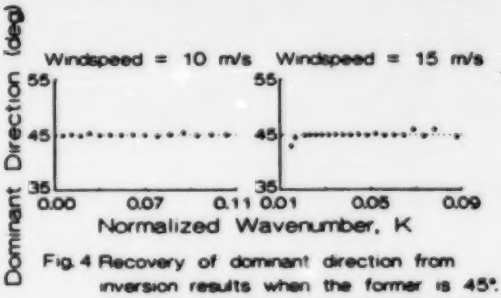

Fig. 4 Recovery of dominant direction from inversion results when the former is 45°. Compare with Fig. 3 to see where K's occur in each relevant spectrum.

Summary

An analysis of how HF backscatter received from the ocean surface by a four-element square array may be used to extract ocean wave parameters has been carried out. The technique was based on a method in [2] in which the wide beam radar cross section is expressed as an angular Fourier series having eight coefficients. These coefficient expressions were reduced to a matrix system which, on inversion by a singular value decomposition algorithm, gives the Fourier coefficients of the ocean wave directional spectrum.

References

[1] D. D. Crombie, Doppler Spectrum of Sea Echo at 13.56 Mc./s., *Nature*, vol. 175, pp 681–682, 1955.

[2] D. E. Barrick and B. J. Lipa, A compact transportable HF radar system for directional coastal wave field measurements, in *Ocean Wave Climate*, M. D. Earle and A. Malahoff (eds.), Plenum, New York, pp 153–201, 1979b.

[3] B. J. Lipa and D. E. Barrick, Extraction of sea state from HF radar sea echo: Mathematical theory and modeling, *Radio Science*, vol. 21, No. 1, pp 81–100, 1986.

[4] E. W. Gill, "An algorithm for the extraction of ocean wave parameters from wide beam HF radar (CODAR) backscatter", *M.Eng. Thesis*, Memorial University of Newfoundland, St. John's, 1990, 112 pp.

[5] G. W. Stewart, *Computer Science and Applied Mathematics*, New York: Academic Press, Inc., 1973, 441 pp.